Starter Question

In a past census of employees, 20% were in favour of a change to working hours. After making changes to staff contracts, the manager now believes that the proportion of staff wanting a change in their working hours has decreased. The manager carries out a random sample of 30 employees, and 2 are in favour of a change in hours.

Stating your hypotheses clearly, test the manager's claim at the 5% level of significance.

11. Hypothesis Testing

p is prop. of employees wanting a change to working hours

H_0: $p = 0.2$

H_1: $p < 0.2$

X = no. of employees in sample in favour of changing hours

Under H_0, $X \sim B(30, 0.2)$

$= 0.05$

$P(X \leq 2) = 0.0442$

$0.0442 < 0.05$

So result is significant. Reject $H_{0.}$

There is sufficient evidence at the 5% level to

0 Statistical hypothesis testing

01	Understand and apply the language of statistical hypothesis testing, developed through a binomial model: null hypothesis, alternative hypothesis, significance level, test statistic, 1-tail test, 2-tail test, critical value, critical region, acceptance region, p-value; extend to correlation coefficients as measures of how close data points lie to a straight line and be able to interpret a given correlation coefficient using a given p-value or critical value (calculation of correlation coefficients is excluded).

Students should:

- recognise whether a given context requires the use of a 1-tail or 2-tail test and understand the difference between them
- be able to state appropriate null and alternative hypotheses to test a population proportion in a given context and know that the null hypothesis always contains the equality sign
- understand that the significance level of a test is the probability of rejecting a correct null hypothesis in error
- be able to find the test statistic, which is the observed number of outcomes of the event
- be able to find the critical region for a 1-tail test, or the critical regions for a 2-tail test, supporting the choice of values in such regions with appropriate binomial probabilities
- know that the critical region consists of the critical values for the test and that if the test statistic lies in the critical region then the null hypothesis is rejected
- know that the acceptance region is the range of possible values, that the discrete random variable can take, that do not lie in the critical region and that if the test statistic lies in the acceptance region that this will lead to the acceptance of the null hypothesis
- be able to use the given p-value corresponding to the test statistic or the given critical value(s), for the relevant significance level of the test, to decide whether to accept or reject the null hypothesis; understand that the p-value should be compared to a binomial distribution critical region with probability equal to or less than the significance level
- know that the precise definition of a p-value in a 2-tailed test varies. It can be defined as the probability calculated from the test statistic or twice that value. In order to circumvent this difficulty, questions will not be asked in which students are required to state the p-value for such a test
- be able to interpret a conclusion in context.

0 Statistical hypothesis testing

Notes

- The conclusion of a hypothesis test is an inference based on evidence and thus students must indicate that there is no certainty in their conclusions. Using the phrase "sufficient evidence to suggest" (qualified with a "not" as applicable) would be a good standard to adopt. There is nothing to be gained by trying to write a conclusion creatively. The final concluding statement of a hypothesis test should always relate back to the context.
- In cases where the null hypothesis is not rejected we would allow an inference of "Do not reject H0" or "Accept H0." Statistical purists will prefer the former.

11.1 Hypothesis Testing

For a one-tailed test the **extreme** results will be on only **one side** of the distribution (depending on whether our observed value is to the **left** or **right** of the **expected** number of successes).

For a two-tailed test we could have extreme results on **both sides** of the distribution (remember the alternative hypothesis is that p is *different* from H_0). Therefore we can have significant values at either tail.

For a two-tailed test we compare our -value to instead of .

11.1 Hypothesis Testing

Example 4

A wildlife photographer is taking photographs of a rare glass frog. He's found over time that the probability that he'll sight a glass frog during any day of searching is 0.05.

He moves to another part of the rainforest where he claims the probability will be different. During his first 6 days searching he spots the frog on 3 of the days. Use a 1% level of significance to test his claim.

11.1 Hypothesis Testing

Example 4

p is the probability that he spots the frog on a particular day.

H_0: $p = 0.05$

H_1: $p \neq 0.05$

X is the number of days he spots the frog in the sample.

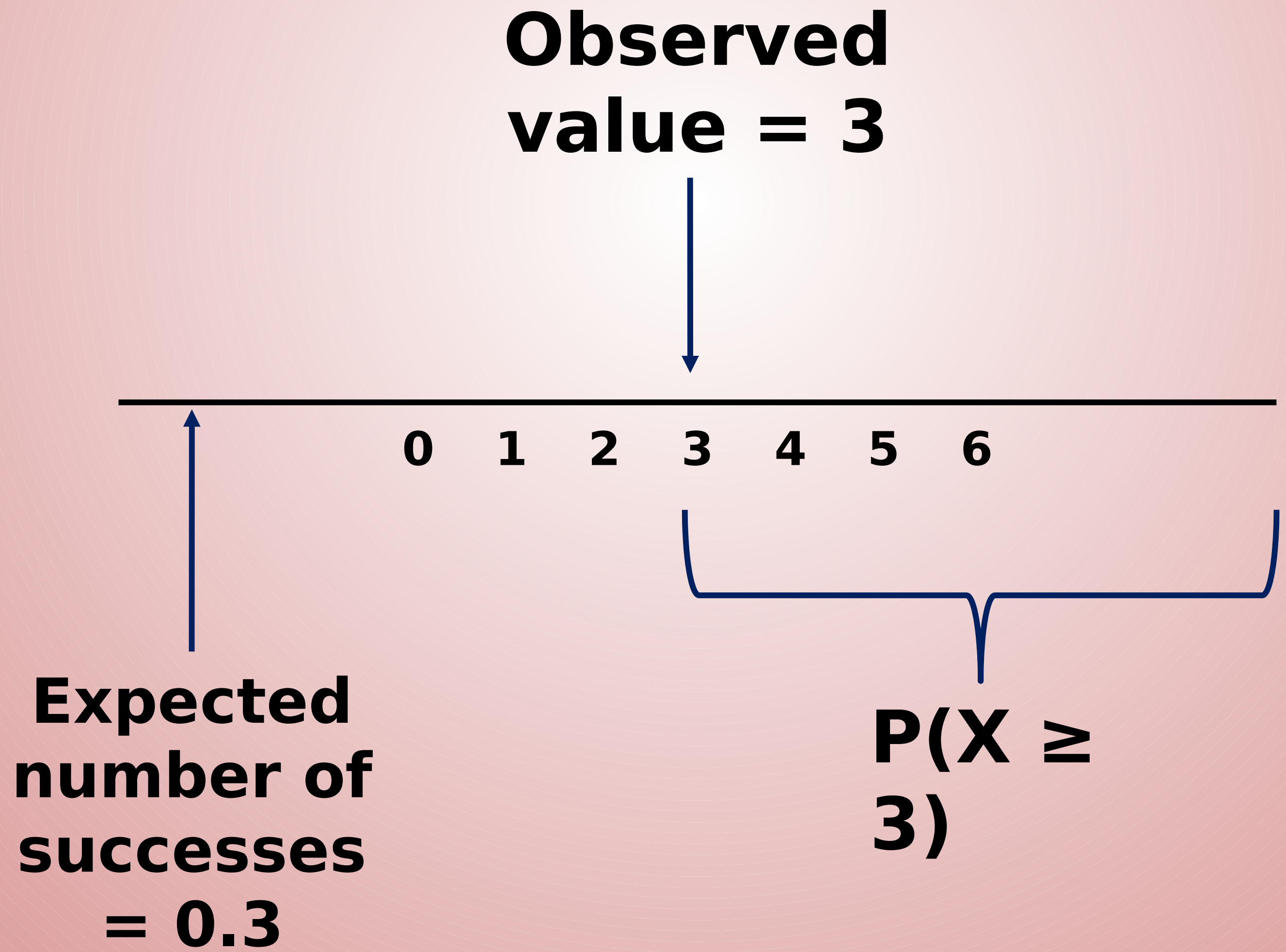
Under H_0, $X \sim B(6, 0.05)$

$= 0.01$, $= 0.005$

$P(X \geq 3) = 1 - P(X \leq 2) = 0.0022$

Since $0.0022 < 0.005$ the result is significant.

Reject H_0. There is sufficient evidence to suggest


Observed value = 3
0 1 2 3 4 5 6
Expected number of successes = 0.3
P(X ≥ 3)

11.1 Hypothesis Testing

Example 5

In the last general election, Party Z won 36% of the vote. An opinion poll company surveys 100 people and finds that 45 support Party Z. Does this provide sufficient evidence, at the 10% level, that the proportion of voters who support Party Z has changed?

p is the proportion of voters supporting Party Z

H_0: $p = 0.36$

H_1: $p \neq 0.36$

X is the number of people in the sample that support Party Z

Under H_0, $X \sim B(100, 0.36)$

11.1 Hypothesis Testing

Example 5

In the last general election, Party Z won 36% of the vote. An opinion poll company surveys 100 people and finds that 45 support Party Z. Does this provide sufficient evidence, at the 10% level, that the proportion of voters who support Party Z has changed?

$P(X \geq 45) = 1 - P(X \leq 44) = 0.0397$

Since $0.0397 < 0.05$ the result is significant.

Reject H_0.

There is sufficient evidence to suggest that the proportion of voters who support Party Z has changed.

11.1 Hypothesis Testing

Example 6

Over a long period of time it has been found that in Enrico's restaurant the ratio of non-vegetarian to vegetarian meals is 2 to 1. In Manuel's restaurant in a random sample of 10 people ordering meals, 1 ordered a vegetarian meal.

Using a 5% level of significance, test whether or not the proportion of people eating vegetarian meals in Manuel's restaurant is different to that in Enrico's restaurant.

e the proportion of people ordering vegetarian meals.

e the number of people ordering vegetarian meals in the sa

11.1 Hypothesis Testing

Example 6

Over a long period of time it has been found that in Enrico's restaurant the ratio of non-vegetarian to vegetarian meals is 2 to 1. In Manuel's restaurant in a random sample of 10 people ordering meals, 1 ordered a vegetarian meal.
Using a 5% level of significance, test whether or not the proportion of people eating vegetarian meals in Manuel's restaurant is different to that in Enrico's restaurant.

Under

Not significant. Do not reject . There is not sufficient evidence to suggest that the proportion of people eating vegetarian meals in Manuel's is different to Enrico's.

11.1 Hypothesis Testing

General Method

1. Define the population parameter in context (Let be...).
2. State the null hypothesis ().
3. State the alternative hypothesis ().
4. State the test statistic (Let be the number of...).
5. Write the probability distribution of (Under ,).
6. State the significance level ().
7. Test for significance (-value) OR find the critical region.
8. Write conclusion in context (is there sufficient evidence to reject).

Exercise 2, 3